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Letter from W. Israel enclosed by JLS.

Vienna, 3rd May 1956

Dear Synge,

Many thanks for your good letter of 25th April. I have not got your book at hand, indeed I have nothing at hand except what we have in the four suitcases (and a few parcels) – and no chance of getting at anything (though it has all arrived safely) before we get a house or a flat – and this seems to be particularly difficult at the moment on account of a preposterous change in the law, dating from 1st January 1956. – But we are now making attempts at 'the top levels', viz. Ministry of Education and the Mayor of Vienna (who is giving me a prize on the 5th inst., great honour and worth about £140 or six weeks bed and breakfast in our present hotel quarters. –

Now, as regards the criticism of your reviewer, I am quite at a loss to understand it. A Lorentz tr. is after all a rotation in x, y, z, ict, and a 4-dimensional rotation amounts to a pair of rotations in two completely orthogonal planes (2-spaces). So I dare stake anything you like on your statement being quite correct. The proof ought not to be long and tedious. However just as I am writing this, I do remember something about an exceptional case. I try to recollect it. A rotation is given (essentially) by an antisymmetric tensor f_{ik}. This can usually be reduced (by change of frame) to

$$\begin{matrix} 0 & f_{12} & 0 & 0 \\ -f_{12} & 0 & 0 & 0 \\ 0 & 0 & 0 & f_{34} \\ 0 & 0 & -f_{34} & 0 \end{matrix}$$

But there is an exception, viz. when both invariants vanish. For this would require, that after reduction

$f_{12} f_{34} = 0 \qquad f_{12}^2 \pm f_{34}^2 = 0,$

hence $f_{12} = f_{34} = 0$ which cannot be.

Is that the 'parabolic' case? And must I eat my words? Maybe. For this degenerate case has probably no meaning for truly euclidean rotations, but has one for pseudo-euclidean rotations. Let us call the f_{ik} by the Maxwellian names E, H. Then a tensor with $(EH)=0$, $E^2 - H^2 = 0$ is quite alright (plane light wave), while with $E^2 + H^2 = 0$ (euclidean case) $E = H = 0$.

What does this case mean? One can reduce to e.g. E_x, H_y. Thus to

$$\begin{pmatrix} 0 & 0 & f_{13} & f_{14} \\ 0 & 0 & 0 & 0 \\ -f_{13} & 0 & 0 & 0 \\ -f_{14} & 0 & 0 & 0 \end{pmatrix}$$

Yes, I have to eat my words. This is a rotation in the (13)-plane <u>and</u> a rotation in the (14)-plane; but with the <u>two</u> angles <u>equal</u>. Strange and interesting. An "ordinary" Lorentz tr. "in the x-direction" combined with a rotation in the xz-plane, with a fixed relation between the two parameters.

Well, — But the case is highly singular. Indeed all f_{ik} of these properties can, in a suitable frame, be approximated to zero as closely as ~~[illegible]~~ desired. I.e. given any finite ε, however small, you can make all the $f_{ik} < \varepsilon$. So I would still maintain that this singular case, though of mathematical interest, is of no particular relevance in physics.

Please remember me to Mrs. Synge and to the crowd at the Institute. And I assure you that "I too am not quite myself without the Institute". People here are unmeasurably kind to me, but it is not the same thing. We had a true family life at the Institute.

Yours very sincerely

Erwin Schrödinger