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## Efficient micromagnetic modelling of spin-transfer torque and spin-orbit torque

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While the spin-diffusion model is considered one of the most complete and accurate tools for the description of spin transport and spin torque, its solution in the context of dynamical micromagnetic simulations is numerically expensive. We propose a procedure to retrieve the free parameters of a simple macro-spin like spin-torque model through the spin-diffusion model. In case of spin-transfer torque the simplified model complies with the model of Slonczewski. A similar model can be established for the description of spin-orbit torque. In both cases the spin-diffusion model enables the retrieval of free model parameters from the geometry and the material parameters of the system. Since these parameters usually have to be determined phenomenologically through experiments, the proposed method combines the strength of the diffusion model to resolve material parameters and geometry with the high performance of simple torque models. © 2017 Author(s). All article content, except where otherwise noted, is licensed under a Creative Commons Attribution (CC BY) license (<http://creativecommons.org/licenses/by/4.0/>). <https://doi.org/10.1063/1.5006561>

### INTRODUCTION

The effect of spin torque has recently gained a lot of interest in the magnetics community due to its possible application in storage and sensor technologies. Devices that exploit spin torque include spin-torque oscillators<sup>1,2</sup> as well as spin torque magnetic random access memory.<sup>3,4</sup> Sources of spin torque are spin polarized conducting electrons that interact with the background magnetization. Two mechanisms that lead to spin torque are depicted in Fig. 1. If the conducting electrons are polarized due to a reference layer that acts as a spin filter, the resulting torque is referred to as spin-transfer torque (STT), see Fig. 1(a). If the conducting electrons are polarized due to spin-orbit induced spin splitting, the resulting torque is referred to as spin-orbit torque (SOT), see Fig. 1(b). Numerical simulations are essential to the development of both STT and SOT devices, either to explain experimental measurements or to guide the design of novel devices. A well established model for the description of magnetization processes on the required length scales is the micromagnetic theory. While classical micromagnetics does not cover spin transport effects, various methods have been proposed to extend this model to include spin torque. A popular method for the description of STT in magnetic multilayers is the model of Slonczewski.<sup>5</sup> This model is both efficient and sufficiently accurate for a certain class of problems. However, it introduces free variables that depend on various system parameters such as geometry and materials. These parameters are usually determined in a phenomenological fashion which prevents the systematic investigation of the influence of geometry and material parameters. A more general model that describes both STT and SOT and that depends on material parameters only is the spin-diffusion model introduced in Ref. 6. While this model outperforms the simple Slonczewski model in many regards, it is computationally expensive since it requires the assembly and solution of a large linear system per time step. In this work we propose a

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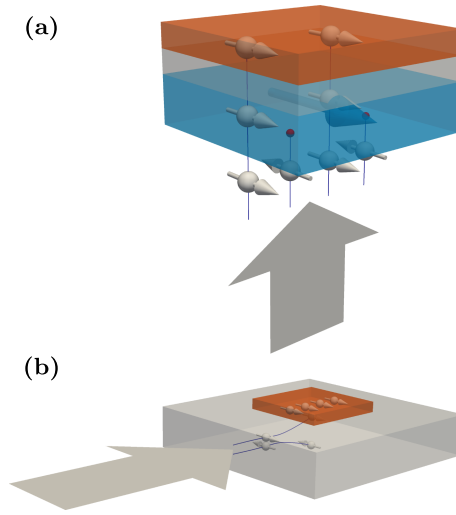


FIG. 1. Sources of spin torque. The large grey arrow marks the moving direction of conducting electrons. The colored areas represent magnetic layers, the grey areas represent metallic layers. (a) Spin-transfer torque. Large blue arrow marks the magnetization direction. (b) Spin-orbit torque.

procedure that uses the spin-diffusion model to obtain the free parameters of a generic torque model, similar to the model of Slonczewski, that works for both STT and SOT.

### SPIN-DIFFUSION MODEL

The normalized magnetization dynamics  $\mathbf{m}(t)$  in micromagnetics is governed by the Landau-Lifshitz-Gilbert equation (LLG)

$$\frac{\partial \mathbf{m}}{\partial t} = -\gamma \mathbf{m} \times \mathbf{h}_{\text{eff}} + \alpha \mathbf{m} \times \frac{\partial \mathbf{m}}{\partial t} + \mathbf{T} \quad (1)$$

with  $\gamma$  being the gyromagnetic ratio,  $\mathbf{h}_{\text{eff}}$  being the effective field, and  $\alpha$  being the damping constant. In case of the spin-diffusion model, the LLG is supplemented by an additional torque term  $\mathbf{T}$  given by

$$\mathbf{T} = -\frac{\gamma J}{\hbar \gamma M_s} \mathbf{m} \times \mathbf{s} \quad (2)$$

where  $J$  is the exchange strength between conducting electrons and magnetization,  $M_s$  is the saturation magnetization, and  $\mathbf{s}$  is the spin accumulation. The spin accumulation describes the spin polarization of the conducting electrons as compared to their polarization in the absence of electric current. Within the spin-diffusion model  $\mathbf{s}$  is solved by the system

$$\nabla \cdot \mathbf{j}_e = 0 \quad (3)$$

$$\nabla \cdot \mathbf{j}_s = -\frac{\mathbf{s}}{\tau_{\text{sf}}} - J \frac{\mathbf{s} \times \mathbf{m}}{\hbar} \quad (4)$$

where  $\tau_{\text{sf}}$  is the spin-flip relaxation time,  $\mathbf{j}_e$  is the vector-valued charge current and  $\mathbf{j}_s$  is the matrix-valued spin current. In the standard diffusion model introduced in Ref. 6 these currents are defined as

$$\mathbf{j}_e = 2C_0 \mathbf{E} - 2D_0 \frac{\beta' e}{\mu_B} (\nabla \mathbf{s})^T \mathbf{m} \quad (5)$$

$$\mathbf{j}_s = 2C_0 \frac{\beta \mu_B}{e} \mathbf{m} \otimes \mathbf{E} - 2D_0 \nabla \mathbf{s} \quad (6)$$

where  $C_0$  is connected to the electric conductivity,  $D_0$  is the diffusion constant, and  $\beta$  and  $\beta'$  are dimensionless polarization parameters. The numerical solution of the system (3)–(6) is described in detail in Ref. 7. While this model accounts for the spin transport between layers due to diffusive

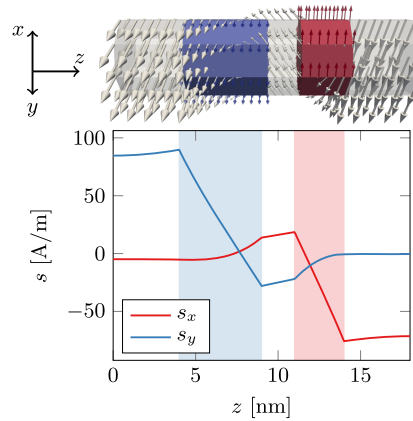


FIG. 2. Spin-accumulation in a magnetic multilayer with pinned layer (blue) homogeneously magnetized in  $y$ -direction and free layer (red) homogeneously magnetized in  $x$ -direction.

processes, it does not account for the spin accumulation generated by spin-orbit interaction. These effects can be included in the spin-diffusion model by modification of the currents according to<sup>8</sup>

$$j'_{e,i} = j_{e,i} - \epsilon_{ijk} \theta \frac{e}{\mu_B} j_{s,jk} \quad (7)$$

$$j'_{s,ij} = j_{s,ij} + \epsilon_{ijk} \theta \frac{\mu_B}{e} j_{e,k} \quad (8)$$

where  $\theta$  is the spin-Hall angle. Here  $j_e$  and  $j_s$  are inserted from (5) and (6) and  $j'_e$  and  $j'_s$  are inserted into (3) and (4).

Figure 2 depicts the spin accumulation  $s$  in a typical STT device with a thick pinned layer that is homogeneously magnetized in  $y$ -direction and a thinner free layer homogeneously magnetized in  $x$ -direction. It should be noted that the spin accumulation is highly non uniform within the magnetic layers. Not only the strength of  $s$  changes rapidly within each layer, but moreover it performs a rotation in the magnetic layers. This rotation is a consequence of the bidirectional coupling of the magnetization  $m$  with the spin accumulation  $s$  that leads to a torque exerted on the spin accumulation. While the spin-diffusion model is a powerful tool that accurately accounts for a number of spin transport effects, it adds a significant overhead to numerical calculations since the system matrices representing (3)–(8) have to be assembled and solved for every time step. In order to solve large systems in reasonable time, it is highly desirable to reduce the complexity of the spin-torque model.

## SIMPLIFIED TORQUE MODEL

Typical layer thicknesses of spin-torque devices are well below the exchange length. In this case the magnetization can be assumed to be homogeneous in the perpendicular direction and hence the torque can be described as averaged over the layer thickness without changing the dynamics of the system. The most generic description of this averaged torque with respect to a normalized reference magnetization  $M$  can be written as  $T = T_{\text{damp}} + T_{\text{field}}$  with

$$T_{\text{damp}} = \eta_{\text{damp}}(\vartheta) \frac{j_e \hbar}{2e\mu_0 M_s t} m \times (m \times M) \quad (9)$$

$$T_{\text{field}} = \eta_{\text{field}}(\vartheta) \frac{j_e \hbar}{2e\mu_0 M_s t} m \times M \quad (10)$$

where we distinguish between two possible flavors of the torque, namely the so-called dampinglike torque  $T_{\text{damp}}$  and the fieldlike torque  $T_{\text{field}}$ , see e.g. Ref. 9. The reference magnetization  $M$  is usually chosen to match the expected spin polarization of the conducting electrons in the considered layer, e.g. the orientation of the pinned-layer magnetization in the case of STT. The prefactor  $\eta$  represents an arbitrary angular dependence with  $\vartheta$  being the angle between the magnetization  $m$  and the reference magnetization  $M$ , and  $t$  is the thickness of the considered layer.

Equations (9) and (10) exactly reproduce the model introduced by Slonczewski for the description of STT. In Ref. 10 it is shown that in the case of a general magnetic multilayer with possibly different free and pinned-layer thicknesses, the angular dependence  $\eta$  of both the dampinglike and fieldlike torque takes the form

$$\eta(\vartheta) = \frac{q^+}{A + B \cos(\vartheta)} + \frac{q^-}{A - B \cos(\vartheta)}. \quad (11)$$

The generalized torque description (9) and (10) is also expected to work for SOT devices. However, as for STT, the free parameters for dampinglike and fieldlike torque cannot be derived from the system parameters in a trivial fashion.

We propose to use the spin-diffusion model as introduced in the preceding section in order to determine the free parameters of the generic torque model by a fitting procedure. Figure 3(a) depicts the fitting result for an asymmetric STT structure with a 5 nm pinned layer and a 3 nm free layer separated by a 2 nm spacer layer. For the parameter fitting, the magnetization in the pinned layer is set homogeneously in  $z$ -direction and the magnetization in the free layer is set homogeneously in the  $xy$ -plane with a tilting angle of  $\vartheta$  to the pinned-layer magnetization. The spin accumulation  $s$  is computed with the spin-diffusion model for various angles  $\vartheta$  and the angular dependencies  $\eta_{\text{damp}}$  and  $\eta_{\text{field}}$  are obtained by spatially averaging the torque given by (2) and projecting onto the generic torque expressions (9) and (10). The free parameters  $q^\pm$ ,  $A$ , and  $B$  of expression (11) are then fitted to the distinct values of  $\eta_{\text{damp}}$  and  $\eta_{\text{field}}$  separately. We use typical values for the system, namely we set  $M_s = 8 \times 10^5$  A/m,  $D_0 = 1 \times 10^{-3}$  m<sup>2</sup>/s,  $\beta = \beta' = 0.8$ ,  $\tau_{\text{sf}} = 32$  fs, and  $J = 5.3 \times 10^{-20}$  J for the magnetic layers,  $D_0 = 5 \times 10^{-3}$  m<sup>2</sup>/s and  $\tau_{\text{sf}} = 1$  ps for the spacer layer and  $D_0 = 5 \times 10^{-3}$  m<sup>2</sup>/s and  $\tau_{\text{sf}} = 12.25$  ps for the leads. The fit of the general Slonczewski model to the angular dependence as computed by the spin-diffusion model shows an almost perfect agreement.

A similar procedure was performed for a system subject to SOT, see Fig. 3(b). Here we consider a metallic underlayer with a thickness of 10 nm and material parameters typical for heavy metals, namely  $D_0 = 5 \times 10^{-3}$  m<sup>2</sup>/s,  $C_0 = 6 \times 10^6$  A/Vm,  $\tau_{\text{sf}} = 2.5 \times 10^{-17}$  s, and  $\theta = 0.2$ . On top of the heavy-metal layer we place a magnetic thin film with a thickness of 2 nm and material parameters

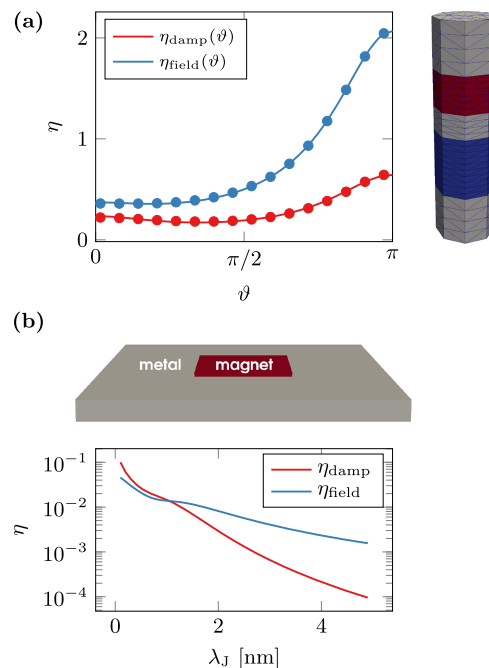


FIG. 3. Angular dependence  $\eta(\vartheta)$  of the simplified spin-torque model fitted to the results of the spin-diffusion model (a) STT in an asymmetric magnetic multilayer fitted the model of Slonczewski. The points mark the simulation results and the lines show the fit. (b) SOT dependence on the diffusion length  $\lambda_{\text{sf}}$  in the magnetic layer.

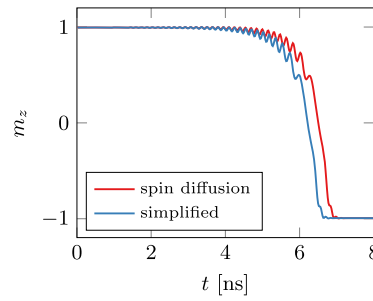


FIG. 4. Current induced switching of the free-layer magnetization in an STT device.

similar to the magnetic layers in the STT structure. A charge current in  $x$ -direction is applied in the metal layer which is expected to generate a spin current in  $z$ -direction with a  $y$ -polarization due to the spin-orbit interaction. With the reference magnetization  $M$  set in  $y$ -direction, the spin-diffusion model reveals a constant prefactor  $\eta$  that does not depend on the angle  $\vartheta$  for both the dampinglike and fieldlike torque. We compute  $\eta_{\text{damp}}$  and  $\eta_{\text{field}}$  for different diffusion lengths  $\lambda_J = \sqrt{2D_0\tau_{\text{sf}}}$  of the magnetic material. The results, depicted in Fig. 3(b), show very similar values of  $\eta_{\text{damp}}$  and  $\eta_{\text{field}}$  for  $\lambda_{\text{sf}}$  with values well below the layer thickness. With increasing  $\lambda_{\text{sf}}$  the fieldlike torque becomes dominant which may be accounted to the fact that the conducting electrons are not able to transfer their complete torque to the magnetization in this case.

## STT SWITCHING

The proposed simplified model is benchmarked with the simulation of an STT switching event. We consider the STT system introduced in the preceding section consisting of a pinned layer and a free layer separated by a conducting spacer. The pinned-layer magnetization is initialized in  $z$ -direction and considered fixed during simulation. In addition to the material parameters introduced in the preceding section, we assume a uniaxial anisotropy with  $K = 10^5 \text{ J/m}^3$  and an axis in  $(0, 0.1, 1)$  direction in the free layer in order to avoid a metastable collinear situation. Moreover we assume an exchange constant  $A = 1.3 \times 10^{-11} \text{ J/m}$  and a damping  $\alpha = 0.02$ . The initial magnetization in the free layer is chosen to point parallel to the anisotropy axis in positive  $z$ -direction. After relaxation of the free layer, a current of  $j_e = 2 \times 10^{11} \text{ A/m}^2$  is applied in positive  $z$ -direction which corresponds to electrons moving in negative  $z$ -direction. The spin torque in this case is expected to lead to a switching of the free-layer magnetization from positive to negative  $z$ -direction.

Fig. 4 shows the switching dynamics computed with the full spin-diffusion model compared to the results of the simplified model. Despite the relatively complex switching behaviour, the simplified model shows a good agreement with the full diffusion model.

## CONCLUSION

Using the spin-diffusion model for the computation of the free parameters of a simplified torque model is a promising approach that enables the efficient simulation of both STT and SOT devices. The presented method confirms the model of Slonczewski as it perfectly reproduces its angular dependence and delivers a straightforward way to retrieve the free parameters of this model that usually have to be determined experimentally. Moreover, the spin-diffusion model reveals the lack of angular dependence of the torque for SOT devices. Compared to the solution of the spin-diffusion model, that requires the assembly and solution of a linear system per timestep, the proposed method is numerically cheap as it only requires a sparse matrix-vector multiplication. With our approach, the full spin-diffusion model has to be solved only a few times for a system with given geometry and materials to obtain the free parameters. The free parameters do not depend on the magnetization configuration of the system and are hence valid throughout dynamical simulations.

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