

Stellar Mass Loss and Cosmic Dust

SE 280460

Unit 3: 9th November 2010

Theory of the interaction between radiation and cosmic dust

- I. The Lorentz oscillator model***
- II. Dielectric functions and optical constants***
- III. Absorption and scattering by small particles***
- IV. The idea of Mie theory***
- V. Application to radiative transfer (Stéphane Sacuto)***

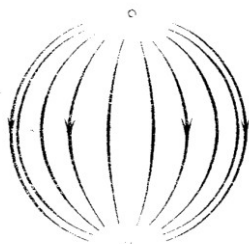
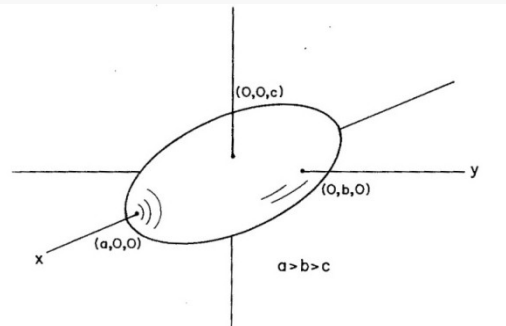
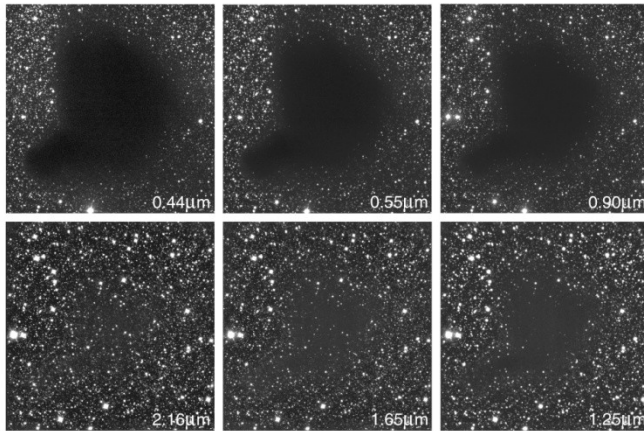


Fig. 3. Erste elektrische Partialschwingung.



$$Q_{\text{abs}} = 4x \operatorname{Im} \frac{\epsilon_l - \epsilon_m}{\epsilon_l + 2\epsilon_m},$$

$$\frac{\langle C_{\text{abs}} \rangle}{V} = \frac{2\pi}{\lambda} \operatorname{Im} \left(\frac{2\epsilon}{\epsilon - 1} \ln \epsilon \right).$$



The Dark Cloud B68 at Different Wavelengths (NTT + SOFI)

ESO PR Photo 29b/99 (2 July 1999)

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Motivation:

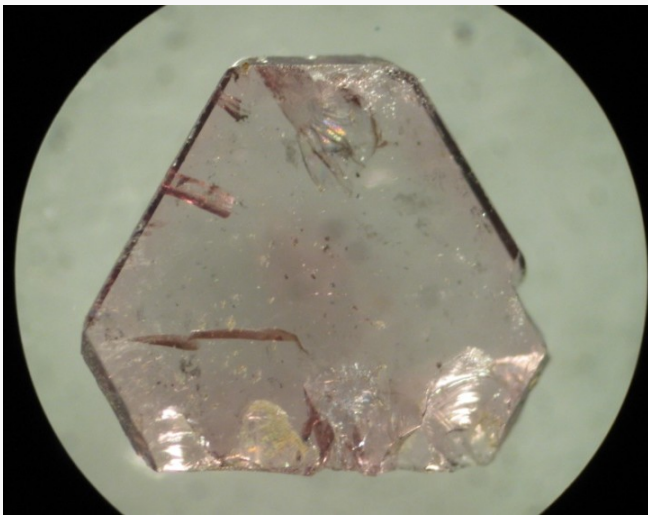
Why do we need abstract concepts like

- Lorentz oscillator model
- Dielectric functions, optical constants
- Absorption and scattering cross sections
- Radiative transfer ... ?

These concepts help us to understand

- Interaction of light with solids in general
- Colours of minerals, glasses, stardust grains:
e.g., why do some crystals appear red?
- Interstellar extinction curve:
e.g., why the $1/\lambda$ – behaviour of absorption?
- Differences between optical properties of crystalline and amorphous solids, carbon and silicates, ...
- How internal structure determines optical properties

< Zeidler et al. (2010), A&A, forthcoming



I. The Lorentz oscillator model

Describing the interaction of electromagnetic waves (or photons) with solids

The Lorentz oscillator model provides a good description of the interaction between photons and electrons / ions in a solid.

The electrons / ions (charge e , mass m) are treated as harmonic oscillators.

The main force acting upon the charged matter elements is the electric component of the electromagnetic radiation (local field $\mathbf{E}$ with circular frequency ω).

$\mathbf{E}$ causes the charged matter elements to oscillate around their equilibrium positions.

The equation of motion can be written as a *linear, inhomogeneous differential equation*:

$$m \mathbf{a} + b \mathbf{v} + K \mathbf{x} = e \mathbf{E} \quad (\text{same equation as for forced damped oscillation})$$

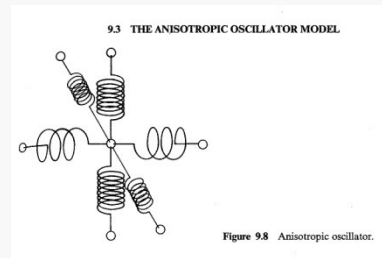
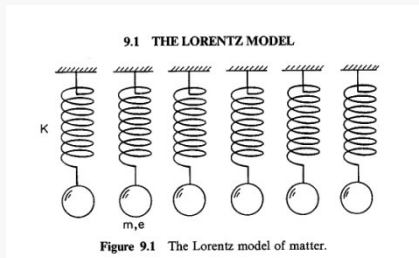
where

$m \mathbf{a}$... mass times acceleration vector

$b \mathbf{v}$... damping term times velocity vector (dissipative term)

$K \mathbf{x}$... “spring constant” (stiffness) times displacement-from-equilibrium-vector

$\mathbf{E} = \mathbf{E}_0 \exp(-i \omega t + \mathbf{k} \cdot \mathbf{r})$... local field vector, oscillating periodically with circular frequency ω .



On the solution of the equation

$$m \ddot{x} + b \dot{x} + K x = e E$$

It has a *transient part* (pure damping, like for a damped oscillator without external force) and an *oscillatory part*, which is the interesting one here. The latter can be written as:

$$x = (e/m) E / RD$$

where

$$RD = \omega_0^2 - \omega^2 - i \gamma \omega \text{ (resonance denominator)}$$

$$\omega_0^2 = K/m \text{ } (\omega_0 \sim \text{resonance frequency})$$

$$\gamma = b/m \text{ (damping constant)}$$

RD is *complex* (if $\gamma \neq 0$) $\leftrightarrow$ proportionality factor between x and E is complex

$\leftrightarrow$ displacement x and local field E are not in phase.

(x may even become anti-parallel to $E \leftrightarrow$ strongest absorption)

Therefore, we may write:

$$x = E (e/m) A e^{i\Theta}$$

$e^{i\Theta}$ has a real and an imaginary part ($\cos \Theta + i \sin \Theta$)

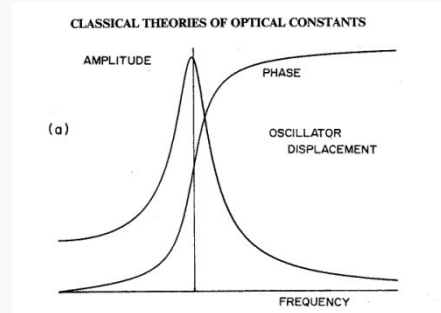
Further discussion of the solution

$$\mathbf{x} = \mathbf{E} (e/m) A e^{i\Theta}$$

We still need expressions for A and Θ :

$$\Theta = \arctan [\gamma \omega / (\omega_0^2 - \omega^2)]$$

$$A = [(\omega_0^2 - \omega^2)^2 + \gamma^2 \omega^2]^{-1/2}$$



Bohren & Huffman (1983),
p. 230

The complex dielectric function ε (also called relative permittivity – since normalized to ε_0) is a proportionality factor between total induced polarization $\mathbf{P}$ in a medium and the local field $\mathbf{E}$ “seen” by the charges:

$$\mathbf{P} = \varepsilon_0 (\varepsilon - 1) \mathbf{E} \quad [= \varepsilon_0 \chi \mathbf{E}, \text{ with } \chi (= \varepsilon - 1) \dots \text{electric susceptibility}]$$

On the other hand, $\mathbf{P} = N e \mathbf{x}$ [with $N \dots$ dipole moment per unit volume]

Therefore:

$$\varepsilon_0 (\varepsilon - 1) = \mathbf{P} / \mathbf{E} = N e \mathbf{x} / \mathbf{E} = N e^2 \mathbf{x} / m = [N e^2 / m] * 1 / R N$$

$$\varepsilon = 1 + [N e^2 / m \varepsilon_0] * [1 / (\omega_0^2 - \omega^2 - i \gamma \omega)]$$

More on the solution in terms of ε on the next slide

Equation for a single Lorentz oscillator model:

For a single harmonic oscillator, the dielectric function ϵ that characterizes the internal field in a medium exposed to a periodic external field (frequency ν , circular frequency $\omega = 2\pi\nu$) can be written as:

$$\epsilon(\omega) = 1 + \frac{N q^2}{m \epsilon_0} \frac{1}{[(\omega_0^2 - \omega^2)^2 + \gamma^2 \omega^2]^{1/2}} e^{i(\arctan(\frac{\gamma \omega}{\omega_0^2 - \omega^2}))}$$

or, equivalently:

$$\epsilon(\omega) = 1 + \frac{N q^2}{m \epsilon_0} \frac{1}{\omega_0^2 - \omega^2 - i \gamma \omega}$$

This leads to the following equations for the real and imaginary parts of ϵ , respectively:

$$\epsilon'(\omega) = 1 + \frac{N q^2}{m \epsilon_0} \frac{\omega_0^2 - \omega^2}{(\omega_0^2 - \omega^2)^2 + \gamma^2 \omega^2}$$

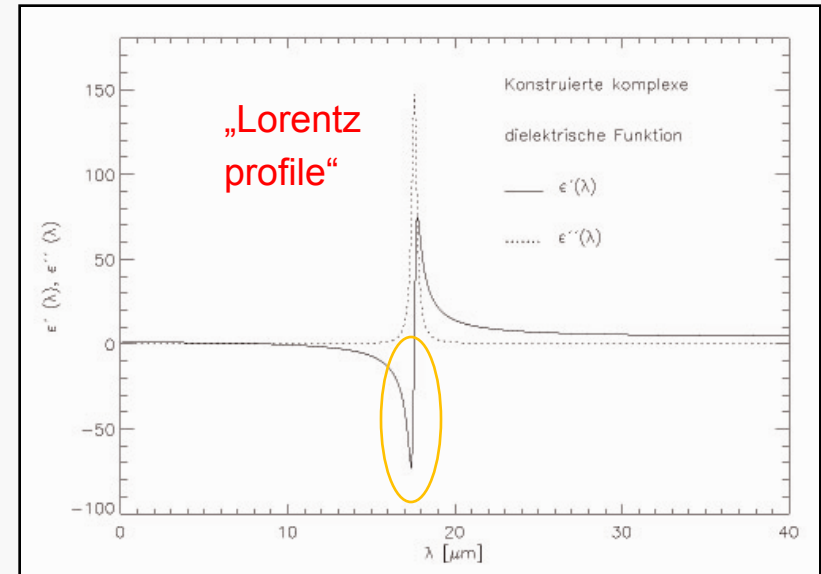
$$\epsilon''(\omega) = \frac{N q^2}{m \epsilon_0} \frac{\gamma \omega}{(\omega_0^2 - \omega^2)^2 + \gamma^2 \omega^2}$$

ϵ'' measures the intensity of absorption of electromagnetic energy (or conversion of this energy into internal energy of the absorbing material).

The meaning of ϵ' is best seen by looking at its behaviour at the resonance frequency. There (where $\omega = \omega_0$), ϵ' undergoes a transition from $\epsilon' < 1$ to $\epsilon' > 1$. This indicates a transition from an induced field antiparallel to the external field to an induced field parallel to the external one. So ϵ' is a measure of the phase shift between internal and external field.

Note:

- (1) The resonance denominator $\omega_0^2 - \omega^2 - i \gamma \omega$ occurs in ϵ again, because it occurred in $\mathbf{x}$
- (2) ϵ'' cannot become negative
- (3) ϵ' can become negative for $\omega > \omega_0$
(it is strongly negative where ϵ'' is very large)



Text after Bohren & Huffman (1983), Graphics from Posch (1998)

Equation for a multiple-Lorentz oscillator model:

$$\epsilon(\omega) = \epsilon_{\infty} + \sum_n \frac{\Delta\epsilon_n \omega_n^2}{\omega_n^2 - \omega^2 - i\gamma_n\omega}$$

where

$\omega = 2\pi\nu$... circular frequency,
 $\nu = c/\lambda$... oscillator frequency,
 ϵ_{∞} ... value of ϵ for $\omega \gg \omega_n \forall n$,
 $\Delta\epsilon_n$... oscillator strength n ,
 ω_n ... eigenfrequency n ,
 γ_n ... damping constant n ,
 n ... oscillator number.

Quantum-mechanical expression
for the dielectric function:

$$\epsilon(\omega) = 1 + \sum_j \frac{(Ne^2/m\epsilon_0)f_{ij}}{\omega_{ij}^2 - \omega^2 - i\gamma_{ij}\omega}$$

where

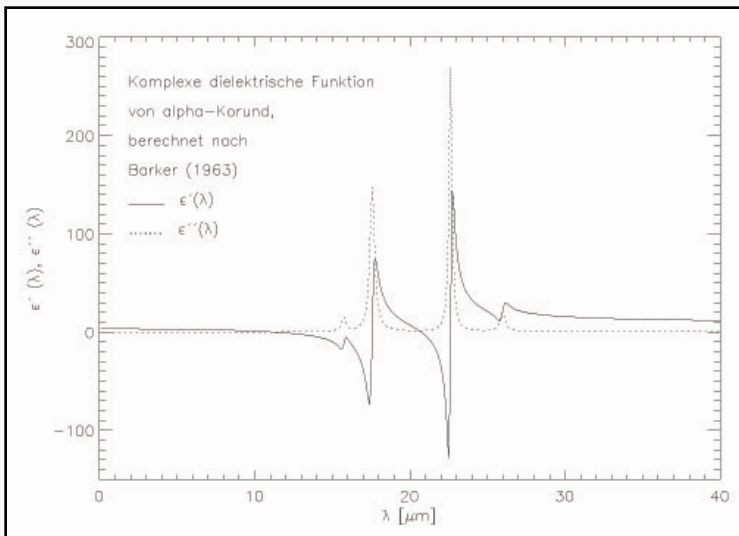
N ... number of quantized oscillators per unit volume,

m, e ... mass, charge of the oscillator (e.g. electron),

f_{ij} ... probability of excitation from state i to state j
(instead of oscillator strength),

ω_{ij} ... energy difference (divided by $\hbar/2\pi$) between the
initial state i and excited state j (instead of eigenfre-
quency),

γ_{ij} ... probability of transition to all other quantum states
(instead of damping constant).



The bottom line:

The four Lorentz oscillator parameters are:

ϵ_{inf} , $\Delta\epsilon_n$, ω_n and γ_n

Knowing these parameters

= knowing the dielectric function of a solid

(if it can be described by the Lorentz osc. model)

What oscillates at which frequency?

- High ω (UV, VIS): ω_0 's of electrons
- Low ω (IR): ω_0 's of atoms / ions
- Still lower ω : "lattice modes" (e.g. layers of the crystal lattice oscillate)

Remember: $\omega_0^2 = K / m$

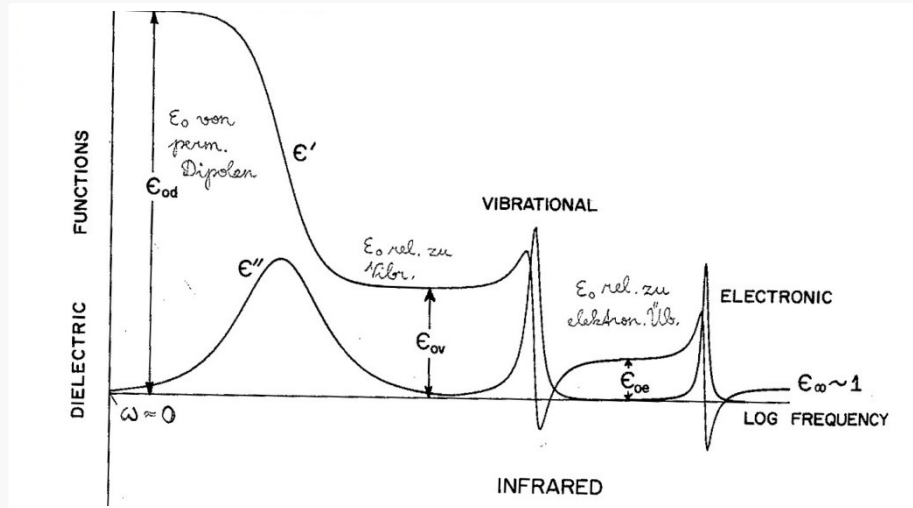
Hence: Larger $m \rightarrow$ smaller ω_0
 ((e.g.: $\sqrt{[m_{\text{proton}}/m_{\text{electron}}]} \sim 40$))

Heavier ions – smaller ω_0 's
 (= larger resonance- λ_0 's)

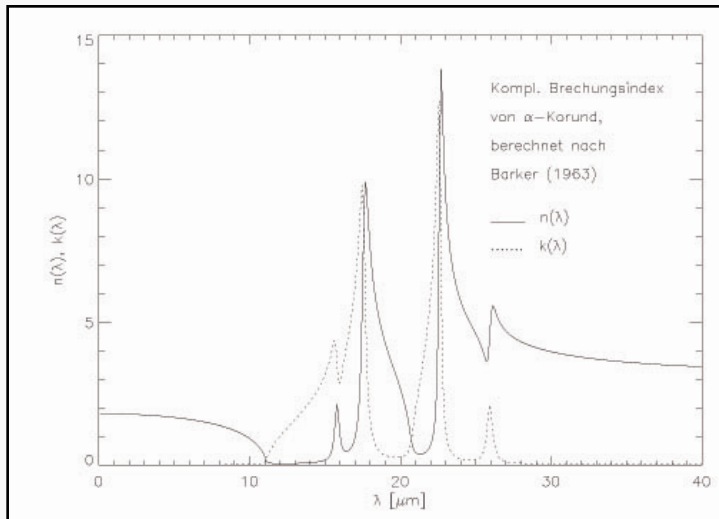
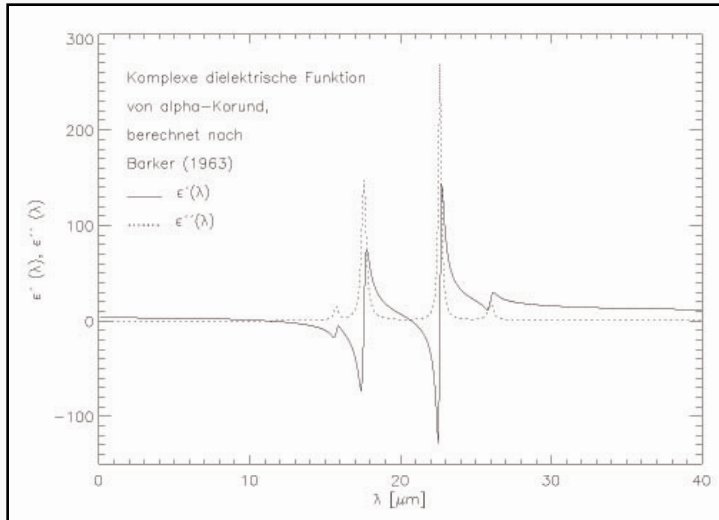
Only few ions produce FIR bands
 Most ionic vibrations correspond to
 resonance- λ 's of 5...50 μm

Electronic transitions – also in solids
 – correspond to the visual and
 UV

Electronic transitions cause colours
 of crystals (and glasses and
 stardust grains)!



Graphics from Bohren & Huffman (1983)



II. Dielectric functions and optical constants

ϵ' and ϵ'' are related to the **optical constants** n and k

For *nonmagnetic* materials ($\mu = \mu_0$) the relation becomes very simple.

It follows from Maxwell's equations for the case of plane-wave propagation that

$$m = n + ik = (\epsilon' + i\epsilon'')^{1/2}$$

$$\epsilon = (n + ik)^2$$

$$\epsilon' = n^2 - k^2 \quad (\text{negative if } k > n)$$

$$\epsilon'' = 2nk \quad (\text{always } > 0)$$

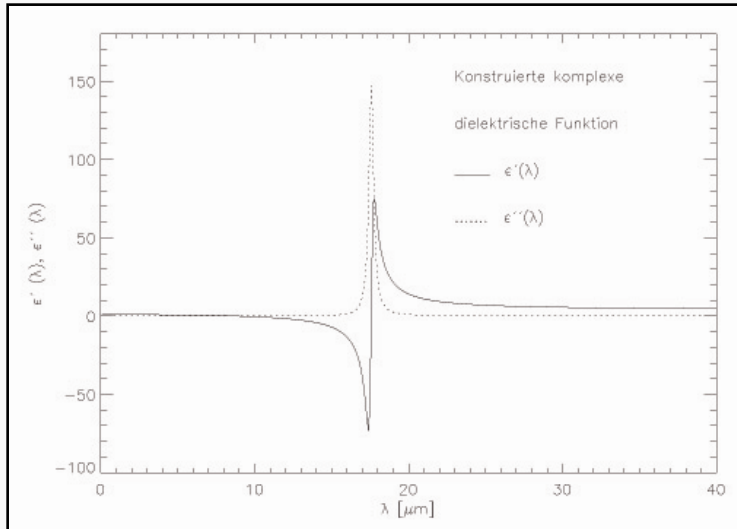
n is known from Snell's refraction law:

$$n = \sin \alpha / \sin \beta$$

k occurs in Beer-Lambert's absorption law:

$$I = I_0 \exp(-4\pi k / \lambda)$$

See Bohren & Huffman, Sects. 2.6, 9.1. Figs from Posch (1998)



Dielectric functions & optical constants (contd.)

The maximum value of ϵ'' is proportional to $1/\gamma \omega_0$

$$\epsilon''_{\text{Max}} \sim [\text{Ne}^2 / m \epsilon_0] \times 1/\gamma \omega_0$$

The width of the ϵ'' – profile is directly proportional to, and even equal to, γ :

$$\text{FWHM}(\epsilon'') = \gamma$$

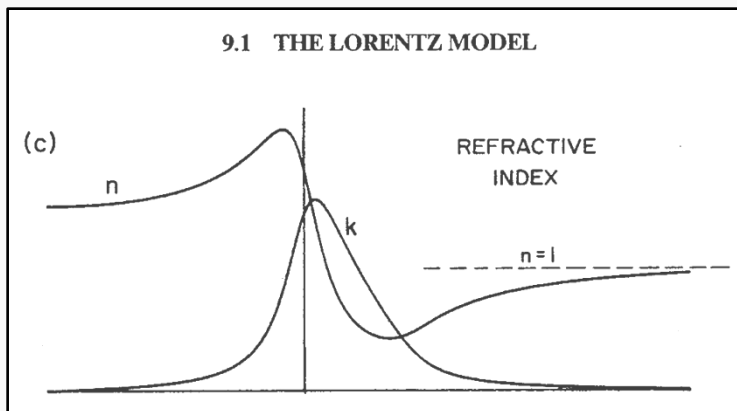
Normal and anomalous dispersion:

“Far away” from the resonance(s),

- n increases with ω
- n decreases with λ
- blue is refracted more strongly than red: this is called “normal dispersion”

In the immediate vicinity of a resonance,

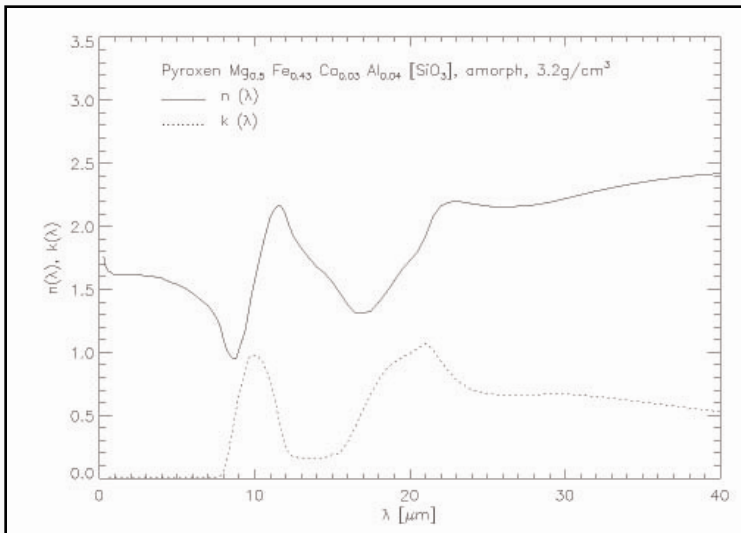
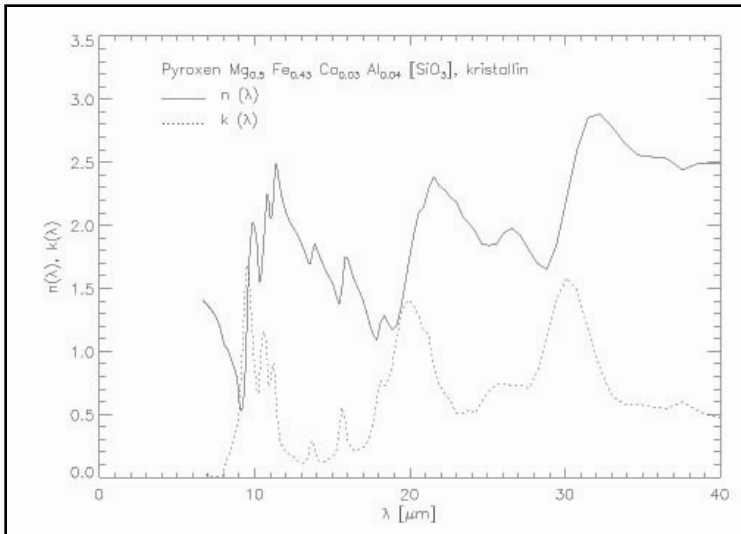
- n decreases with ω
- n increases with λ
- this is called “anomalous dispersion”



Lower Fig. and text: Bohren & Huffman (1983), Sect. 9.1

Remarks on the optical constants

- In vacuum, $n = 1$ and $k=0$
- For absorbing solids, k 's order of magnitude may strongly vary with ω (or with λ):
from 10^{-10} (and less) to 10 (and more)
- The $k(\omega)$ or $k(\lambda)$ function tells a lot about absorption mechanisms that are at work in a solid (electronic, ionic, lattice modes)
- Understanding $k_{\text{vis}}(\lambda)$ for a solid = understanding its colour(s)
- As for n , its deviation from 1 (not from 0) is the interesting aspect
- n can be < 1 (at strong resonances)
- This implies $c/n > c$
- But this merely concerns the phase velocity, not the signal velocity in a medium



Bohren & Huffman, Sect. 9.1.3
 Posch (1998)

How to determine the optical constants

- Will be discussed in greater detail in Unit 5
- $k(\lambda)$: E.g. From transmission measurements using (thin) sections of solids, especially in the UV/VIS.
Based on $T(\lambda) \sim \exp(-4\pi k / \lambda)$

- $n(\lambda)$ and $k(\lambda)$: May both be derived from reflectance measurements at normal incidence, using polished surfaces

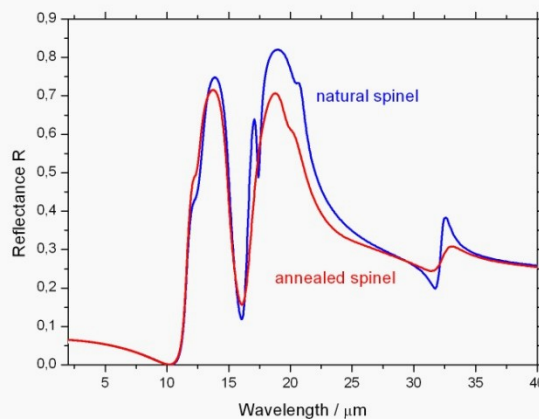
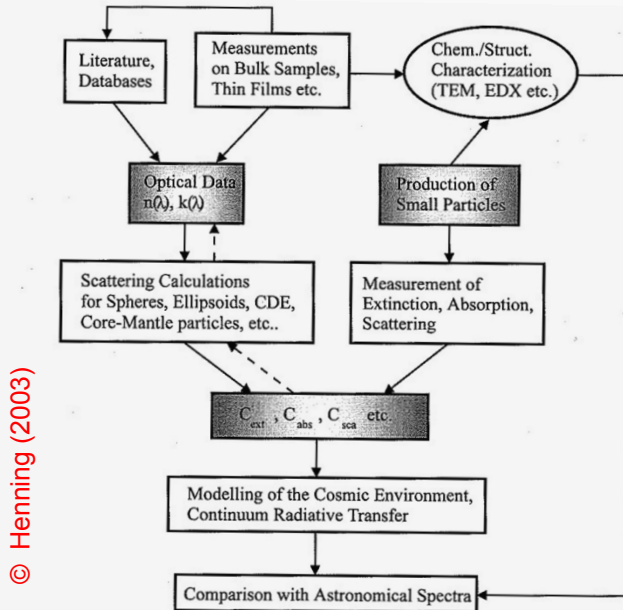
Based on $R(\lambda) = |(m-1)/(m+1)|^2$

corresp. to $R(\lambda) = [(n-1)^2 + k^2] / [(n+1)^2 + k^2]$

- It is possible to derive both $n(\lambda)$ and $k(\lambda)$ from the reflectance (unless $k \sim 0$) since they are internally related to each other by the Kramers-Kronig-Relations

- One standard procedure of deriving $n(\lambda)$ and $k(\lambda)$ from $R(\lambda)$ is called “Lorentz oscillator fit”:

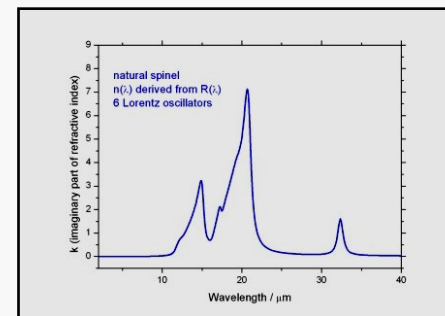
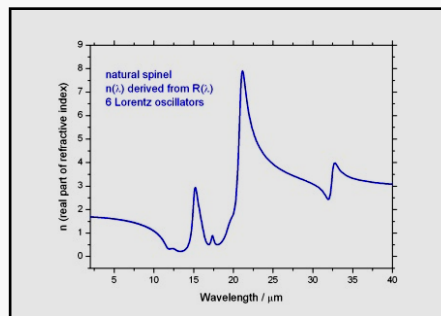
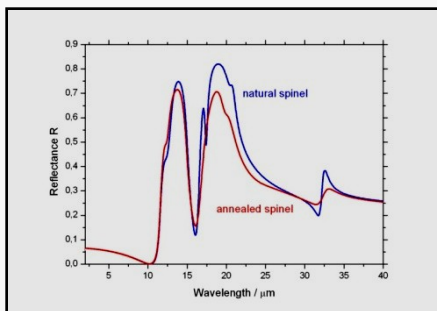
Adjusting ϵ_{inf} , $\Delta\epsilon_n$, ω_n and γ_n to a measured $R(\lambda)$ curve



A typical set of Lorentz oscillator parameters ...

... derived from $R(\lambda)$ measurements of a sample of spinel (MgAl_2O_4)

j	ω_j [1/cm]	$\Delta\epsilon_j$ [1/cm]	γ_j [1/cm]
1	803.6	159.7	82.5
2	666.6	522.9	31.9
3	577.3	132.1	11.5
4	492.4	660.2	30.8
5	478.6	655.5	14.0
6	308.1	151.0	6.9



Is it really worth going back to the dielectric functions?



Wouldn't it be sufficient – for astrophysical applications – to “measure” $n(\lambda)$ and $k(\lambda)$ and start from there?

We do need $\varepsilon'(\lambda)$ and $\varepsilon''(\lambda)$ to make sense of the Lorentz oscillator parameters – and we do need the Lorentz oscillator parameters to

- characterize the optical & dielectric properties of solids with a small set of basic parameters
- calculate $\varepsilon'(\lambda)$, $\varepsilon''(\lambda)$, $n(\lambda)$, $k(\lambda)$ – and much more – for any desired λ -grid
- understand changes in a measured spectrum, e.g., $R(\lambda)$, due to physical effects such as increased damping
- relate IR spectra to the theory of lattice structures (e.g. theory of space groups)
- verify if you have measured (all) the predicted IR active modes of a solid

III. Absorption and scattering by small particles

ε' , ε'' , n and k characterize a material – regardless of geometrical properties.

To describe small particles with given shapes and sizes, other quantities are better suited:

$Q_{\text{abs}}(\lambda)$, $Q_{\text{sca}}(\lambda)$... the absorption and scattering efficiencies

These crucially depend on particle sizes and –shapes (in addition to the material properties)

$Q_{\text{abs}}(\lambda)$ tells us how much radiation is transformed into internal energy (and re-emitted at another wavelength).

$Q_{\text{sca}}(\lambda)$ tells us how much radiation goes back into space without frequency change – but usually into other directions.

The sum of both quantities is the extinction efficiency: $Q_{\text{ext}} = Q_{\text{abs}} + Q_{\text{sca}}$

The following is known as albedo (“whiteness”): $A = Q_{\text{sca}} / Q_{\text{ext}}$ (= 100% for $Q_{\text{abs}} = 0$)

Why the sky is blue: the Rayleigh limit of Mie theory

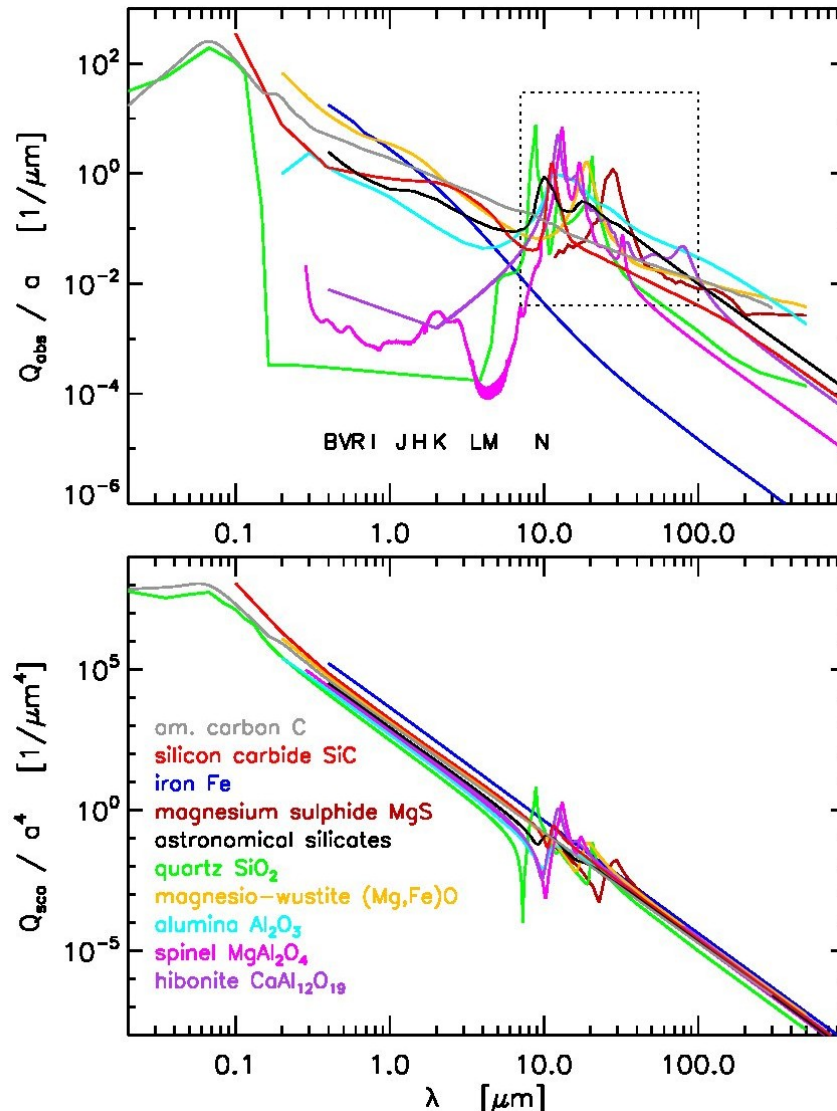
For spherical particles small compared to λ (size parameter $x = [2 \pi a / \lambda] \ll 1$), we find:

$$Q_{\text{abs}} = 4x \operatorname{Im} \frac{\varepsilon_l - \varepsilon_m}{\varepsilon_l + 2\varepsilon_m},$$

$$Q_{\text{sca}} = \frac{8}{3} x^4 \left| \frac{\varepsilon_l - \varepsilon_m}{\varepsilon_l + 2\varepsilon_m} \right|^2,$$

ε_m ... dielectric function of surrounding medium
if vacuum: $\varepsilon_m = 1 + 0i$

The factor „2“ only applies to spherical grains! Different for ellipsoids



< Examples for $Q_{\text{abs}}(\lambda)$, $Q_{\text{sca}}(\lambda)$ for spheres

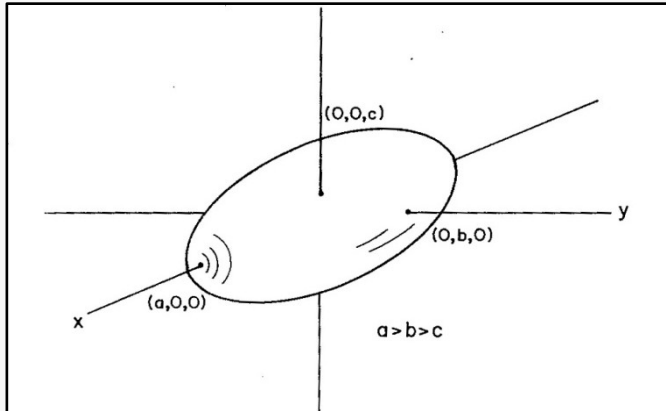
- $Q_{\text{abs}}(\lambda) \sim \lambda^{-p}$ ($p=1\dots 2$) (for $\lambda \gg a$)
- $Q_{\text{sca}}(\lambda) \sim 1/\lambda^4$ (for $\lambda \gg a$)
- Resonance condition for spheres:
 $\epsilon = -2\epsilon_m$, which means, if $\epsilon_m = 1$:
 $\epsilon' = n^2 - k^2 = -2$ and $\epsilon'' = 2nk \sim 0$
 $\rightarrow n \sim 0$; $k \sim \sqrt{2}$

Most materials show resonances
 Other do not (e.g. Fe, amorphous C) – why?

This depends on whether a dipole moment can be induced or not – which depends on whether we have a monoatomic solid or not

Fe and **amorphous C** are monoatomic
 all other examples: a least diatomic
 (situation changes if impurities occur)

© Bohren & Huffman (1983)



What to do if the particles are nonspherical?

Dust grains are not spheres. But they may be approximated by (ensembles of) spheres or by (ensembles of) ellipsoids.

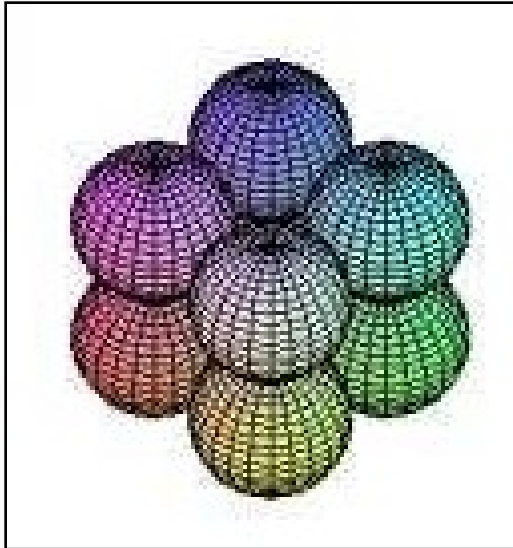
An important special case: the *continuous distribution of ellipsoids (CDE)*.

If all axis ratios are equally probable, then:

$$\frac{\langle C_{\text{abs}} \rangle}{V} = \frac{2\pi}{\lambda} \text{Im} \left(\frac{2\epsilon}{\epsilon - 1} \ln \epsilon \right).$$

C_{abs}/V is equivalent to Q_{abs}/a .

© Anja Andersen (2005)



Ensembles of spheres can be treated by the „DDA“ = discrete dipole approximation.

A commonly used Fortran Code for this case is

DDSCAT (vs. 7.1) by Bruce Draine

→ <http://www.astro.princeton.edu/~draine/DDSCAT.html>

Electrostatic approximation
Mode of homogeneous polarization

380

G. Mie.

system sei ein Rechtssystem. Es sei ferner der Radiusvektor eines Punktes (x, y, z) mit r bezeichnet, der Winkel, den r mit der Achse 1 bildet, sei ϑ , der Winkel der Projektion von r auf die $(2, 3)$ -Ebene mit der Achse 2 sei φ (Fig. 1). Dann sind also (r, ϑ, φ) Polarkoordinaten des Punktes. Wir wollen ferner die Komponenten der elektrischen und der magnetischen Feldintensität in diesem Polarkoordinatensystem bezeichnen mit $\mathfrak{E}_r, \mathfrak{E}_\vartheta, \mathfrak{E}_\varphi$ und $\mathfrak{H}_r, \mathfrak{H}_\vartheta, \mathfrak{H}_\varphi$. Es läßt sich dann leicht zeigen, daß die Maxwell'schen Gleichungen die folgende Form annehmen:

Fig. 1. Koordinatensystem.

$$(1) \left\{ \begin{array}{l} r^2 \cdot \sin \vartheta \left(k \cdot \frac{\partial \mathfrak{E}_r}{\partial t} + A \cdot \mathfrak{E}_r \right) = \frac{\partial (r \cdot \sin \vartheta \cdot \mathfrak{H}_\vartheta)}{\partial \vartheta} - \frac{\partial (r \cdot \mathfrak{H}_\varphi)}{\partial \varphi}, \\ r \cdot \sin \vartheta \cdot \left(k \cdot \frac{\partial \mathfrak{E}_\vartheta}{\partial t} + A \cdot \mathfrak{E}_\vartheta \right) = \frac{\partial \mathfrak{H}_r}{\partial \varphi} - \frac{\partial (r \cdot \sin \vartheta \cdot \mathfrak{H}_\varphi)}{\partial r}, \\ r \cdot \left(k \cdot \frac{\partial \mathfrak{E}_\varphi}{\partial t} + A \cdot \mathfrak{E}_\varphi \right) = \frac{\partial (r \cdot \mathfrak{H}_\vartheta)}{\partial r} - \frac{\partial \mathfrak{H}_r}{\partial \vartheta}, \\ -r^2 \cdot \sin \vartheta \cdot \mu \cdot \frac{\partial \mathfrak{H}_r}{\partial t} = \frac{\partial (r \cdot \sin \vartheta \cdot \mathfrak{E}_\vartheta)}{\partial \vartheta} - \frac{\partial (r \cdot \mathfrak{E}_\varphi)}{\partial \varphi}, \\ -r \cdot \sin \vartheta \cdot \mu \cdot \frac{\partial \mathfrak{H}_\vartheta}{\partial t} = \frac{\partial \mathfrak{E}_r}{\partial \varphi} - \frac{\partial (r \cdot \sin \vartheta \cdot \mathfrak{E}_\varphi)}{\partial r}, \\ -r \cdot \mu \cdot \frac{\partial \mathfrak{H}_\varphi}{\partial t} = \frac{\partial (r \cdot \mathfrak{E}_\vartheta)}{\partial r} - \frac{\partial \mathfrak{E}_r}{\partial \vartheta}. \end{array} \right.$$

Hier bedeuten k, A, μ Dielektrizität, Leitvermögen, Permeabilität, alle in demselben Maßsystem. In einem nicht leitenden Medium ist $k \cdot \mu = 1/v^2$, wenn v die Geschwindigkeit elektromagnetischer Wellen in dem Medium ist.

Man könnte nun leicht durch geeignete Eliminationen für $\mathfrak{E}_r$ und $\mathfrak{H}_r$ je eine Differentialgleichung zweiter Ordnung aufstellen, die die übrigen Unbekannten nicht mehr enthält. Wir wollen das aber erst ausführen, nachdem wir die Gleichungen speziell für das Problem regelmäßiger Schwingungen umgeformt haben.

IV. The idea of Mie theory

Gustav Mie, in 1908, calculated an exact solution for the interaction between el.-mag. waves and *spherical particles of arbitrary size*.

He solved Maxwell's equations for this case, using polar coordinates.

The solution involves the *Riccati-Bessel functions* (= modified spherical Bessel functions).

The essential point of the solution is the development into a series of partial waves.

Die von einer kleinen Kugel reflektierte Strahlung setzt sich im wesentlichen stets aus einer endlichen Zahl von Partialwellen zusammen, aber die Zahl der Partialwellen nimmt zu, wenn die Kugel größer wird.

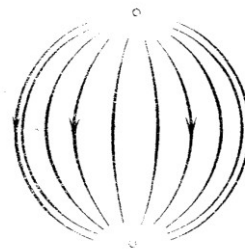


Fig. 3. Erste elektrische Partialschwingung.

From Mie (1908)

1st Partial wave = „mode of uniform polarization“

corresponds to electrostatic approximation

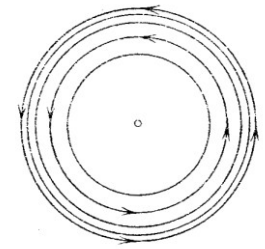


Fig. 4. Erste magnetische Partialschwingung.

From Mie (1908)

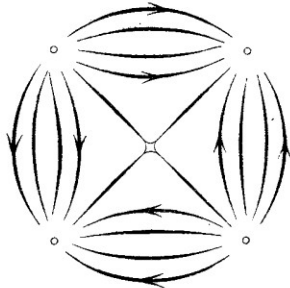


Fig. 5. Zweite elektrische Partialschwingung.

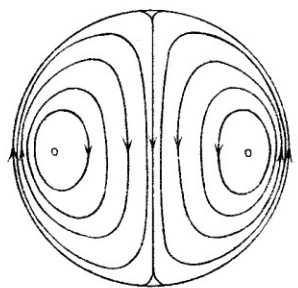


Fig. 6. Zweite magnetische Partialschwingung.

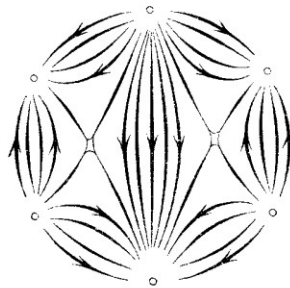


Fig. 7. Dritte elektrische Partialschwingung.

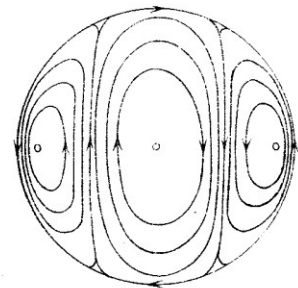


Fig. 8. Dritte magnetische Partialschwingung.

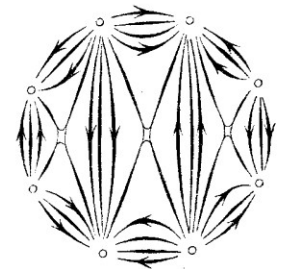


Fig. 9. Vierte elektrische Partialschwingung.

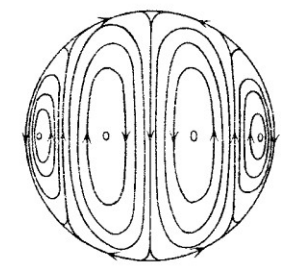


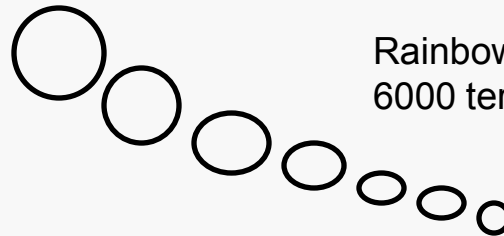
Fig. 10. Vierte magnetische Partialschwingung.

From Mie (1908)

Even though Mie theory involves rather complex mathematics and physics, its formalism can be implemented in an only 167 lines Fortran-Code.

For many cases, the first few terms of the series for $Q_{\text{abs}}(\lambda)$ and $Q_{\text{sca}}(\lambda)$ are sufficient.

Sehr kleine Teilchen strahlen seitlich stets nur die Rayleighsche Welle aus, bei den ganz groben Teilchen in kolloidaler Lösung kommt noch die zweite elektrische und die erste magnetische Partialschwingung hinzu.



Rainbow with $\sim 1\text{mm}$ (1000λ) drops:
6000 terms must be added up!

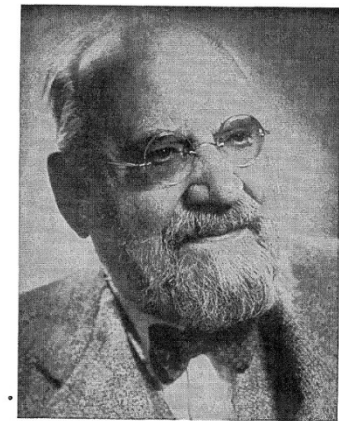


Fig. 1. Gustav Mie (photograph courtesy of H. Spehl, Albert-Ludwigs-Universität).

Gustav Mie (1868-1957), professor of physics in Greifswald, Halle and Freiburg.

His paper from 1908 was entitled „Beiträge zur Optik trüber Medien, speziell kolloidaler Metallösungen“

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G. Mie.

In dem Zahlenbeispiel, das ich unten bringe, um der Erklärung der optischen Eigenschaften kolloidaler Metallösungen näher zu kommen, werde ich bis zu einem Wert des Durchmessers $2\rho = 180 \mu\mu$ gehen. Dieser Wert liegt schon an der Grenze der mikroskopischen Auflösbarkeit, entspricht also wohl der äußersten Größe der Teilchen kolloidaler Lösungen. Auch bei dem großen Durchmesser $2\rho = 180 \mu\mu$, sind schon von $v = 3$ an alle höheren „elektrischen Partialschwingungen“ gegen die beiden ersten ganz verschwindend klein, wir brauchen also nur a_1 und a_2 zu berechnen.

Dagegen müßte man z. B. bei dem Regenbogenproblem, das man auch mit der hier angewendeten Methode in Angriff nehmen könnte, eine ziemlich große Zahl Partialwellen berücksichtigen und würde auf sehr große rechnerische Schwierigkeiten stoßen.

Die beiden ersten Koeffizienten sind:

$$(62) \quad \begin{cases} a_1 = 2 \cdot \alpha^3 \cdot u_1 \cdot \frac{m'^2 - v_1}{m'^2 + 2w_1}, \\ a_2 = -\frac{1}{6} \cdot \alpha^5 \cdot u_2 \cdot \frac{m'^2 - v_2}{m'^2 + \frac{3}{2}w_2}. \end{cases}$$

Wird α sehr klein, so kann man die u, v, w gleich 1 rechnen und bekommt:

$$(63) \quad \begin{cases} a_1 = 2 \cdot \alpha^3 \frac{m'^2 - 1}{m'^2 + 2}, \\ a_2 = -\frac{1}{6} \alpha^5 \frac{m'^2 - 1}{m'^2 + \frac{3}{2}}. \end{cases}$$

Natürlich ist für kleine α auch schon a_2 gegen a_1 verschwindend klein, und es bleibt nur jene erste Partialwelle übrig, deren Existenz zuerst von Lord Rayleigh theoretisch bewiesen worden ist, die ich deswegen manchmal auch kurz die Rayleighsche Strahlung nennen werde.

Modifying the Rayleigh limit

Again, Mie's basic insight in words:

Die von einer kleinen Kugel reflektierte Strahlung setzt sich im wesentlichen stets aus einer endlichen Zahl von Partialwellen zusammen, aber die Zahl der Partialwellen nimmt zu, wenn die Kugel größer wird.

How translate this into equations? To 4th order terms in x , Q_{ext} and Q_{sca} are, after Mie:

$$\begin{aligned} Q_{\text{ext}} &= 4x \operatorname{Im} \left\{ \frac{m^2 - 1}{m^2 + 2} \left[1 + \frac{x^2}{15} \left(\frac{m^2 - 1}{m^2 + 2} \right) \frac{m^4 + 27m^2 + 38}{2m^2 + 3} \right] \right\} \\ &\quad + \frac{8}{3} x^4 \operatorname{Re} \left\{ \left(\frac{m^2 - 1}{m^2 + 2} \right)^2 \right\}, \\ Q_{\text{sca}} &= \frac{8}{3} x^4 \left| \frac{m^2 - 1}{m^2 + 2} \right|^2, \end{aligned}$$

The consequence for small spheres is...:

$$\epsilon = - \left(2 + \frac{12}{5} x^2 \right) \epsilon_m.$$

... instead of $\epsilon = -2\epsilon_m$

... and this means in most cases - if ϵ' increases with frequency near the resonance - **a shift of the small-sphere-resonance to smaller ω (larger λ) with increasing grain size.**

What is the link to astronomical spectra?

Lab measurements and calculation:

- 1) Measure Reflectance $R(n, k, \lambda) \Rightarrow n + i k(\lambda)$
- 2) Measure Transmission $T(n, k, \lambda, F) \Rightarrow k_{\text{NIR}}(\lambda)$
- 3) Calculate $Q_{\text{abs}}(n, k, a, F, \lambda)$

This requires:

- ✓ Preparation of lab analog materials
- ✓ Measure their chemical composition

Astronomical Observations

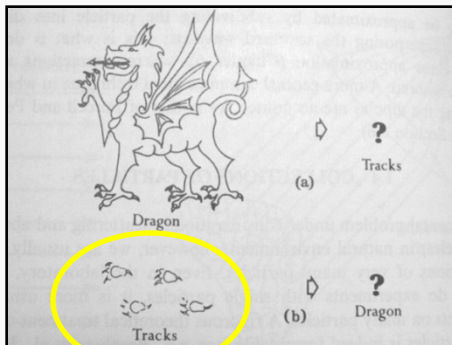
deliver something like:

$$\sum_j Q'_{\text{abs}, j}(n', k', a', F', \lambda) * B_{\lambda}(<T_d>)$$

[more precisely: result of radiative transfer into which $Q_{\text{abs}, i}$ enter]

$\Rightarrow$ Try to constrain $Q'_{\text{abs}, j}(n', k', a', F', \lambda)$ by observations

Comparison



a ... particle 'radius'
 B_{λ} ... Planck function
 F ... Particle shape factor
 $<T_d>$: mean dust temperature

Schedule and topics of the individual seminar sessions:

<i>Unit</i>	<i>Date</i>	<i>Topic</i>
(U1)	05.10.2010	Introduction
(U2)	12.10.2010	Empirical evidences for cosmic dust
(U3)	09.11.2010	Theory of radiation from cosmic dust
(U4)	16.11.2010	Dust formation and processing: How and Where
(U5)	23.11.2010	Studying dust analogs in the laboratory
(U6)	30.11.2010	Dust in stellar outflows
(U7)	07.12.2010	Dust around AGB stars (1)
(U8)	14.12.2010	Dust around AGB stars (2)
(U9)	11.01.2011	Dust in protoplanetary disks
(U10)	18.01.2011	Dust in the Interstellar Medium
(U11)	25.01.2011	Dust in the solar system

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